the returned power $\overline{P_r}$ from scatterers of known $|K|^2$ located within a resolution volume $\mathscr{V}_{\text{res}}$ centered at a range r from a radar yields the *radar reflectivity factor*

$$Z \equiv \frac{1}{\mathscr{V}_{\text{res}}} \sum D^6 = \frac{r^2 \overline{P_r} C_R}{|K|^2} \tag{4.3}$$

where

$$C_R = \frac{64\lambda^2 r^2}{P_t G^2 \pi^2 \mathscr{V}_{\text{res}}} \tag{4.4}$$

and

$$\mathscr{V}_{\text{res}} = \pi\theta_H\theta_V\left(\frac{r}{2}\right)^2 \frac{c_o\tau_p}{2} \tag{4.5}$$

The term ΣD^6 in (4.3) is the sum of the sixth powers of the diameters of all of the scatterers in the volume $\mathscr{V}_{\text{res}}$. Note that since $\mathscr{V}_{\text{res}}$ is proportional to r^2, C_R is a constant depending only on the characteristics of the particular set of radar equipment being used.

Usually there is no way to be certain of the value of $|K|^2$. The scatterers could be composed of liquid, ice, melting ice, insects, turbulent eddies, or chaff. The convention therefore is to use the measured $\overline{P_r}$ and r and (4.3) to calculate the *equivalent radar reflectivity factor*,

$$Z_e \equiv \frac{r^2 \overline{P_r} C_R}{0.93} \tag{4.6}$$

where 0.93 is the value of $|K|^2$ for liquid water. Z_e thus would be the value that the reflectivity factor of the particles producing the returned power $\overline{P_r}$ detected at range r would have if they were composed purely of liquid water. The value of $|K|^2$ for ice is usually set to 0.197. If it were known that the reflectors were actually ice particles, then the true reflectivity factor could be obtained simply by multiplying Z_e by the ratio (0.93)/(0.197). However, since the composition of the scatterers is not normally known with certainty, radar data are usually expressed as Z_e, in units of $\text{mm}^6\ \text{m}^{-3}$, which refer to particle size to the sixth power per unit volume of air. The Z_e values are often given in decibel units with respect to $1\text{mm}^6\ \text{m}^{-3}$:

$$\text{dBZ}_e \equiv 10\log_{10} Z_e \tag{4.7}$$

Typical values of $\text{dB}[Z_e(\text{mm}^6\ \text{m}^{-3})/(1\text{mm}^6\ \text{m}^{-3})]$ are ~−30 to 0 in marginally detectable precipitation; ~0–10 in drizzle, very light rain, or light snow; ~10–30 in moderate rain and heavier snow; ~30–45 in melting snow; ~30–60 in moderate to heavy rain; and ~60–70 or more in hail.

4.2.2 Relating Reflectivity to Precipitation

One of the most important uses of meteorological radar is to estimate the precipitation content of the air, the precipitation rate, and the fall speed of the precipitation at the earth's surface. In this section, we will outline the techniques used to make these estimates from measurements of radar reflectivity.

4.2.2.1 Particle Size Method

Since the radar reflectivity factor Z is related to particle size, there is a physical basis for a quantitative relationship between the precipitation content of the air and the received radar echo intensity. According to the definition (4.3), the radar reflectivity factor is the sixth moment of the particle size distribution. If the number of particles in a radar-sampled volume is very large, a continuous version of (4.3) can be used:

$$Z = \int_0^\infty D^6 N(D)\, dD \tag{4.8}$$

where $N(D)$ is the particle size distribution function defined such that $N(D)dD$ is the number of particles of diameter D to $D + dD$ per unit volume of air. If the scatterers are liquid water, $Z = Z_e$. The particle size distribution also determines the mixing ratio of rainfall,

$$q_r = \frac{\pi\rho_L}{6\rho}\int_0^\infty D^3 N(D)\, dD \tag{4.9}$$

where ρ_L is the density of liquid water, ρ is the density of air, and the rainfall rate $\mathfrak{R}$ is given by

$$\mathfrak{R} = \frac{\pi\rho_L}{6}\int_0^\infty V(D) D^3 N(D)\, dD \tag{4.10}$$

where $V(D)$ is the fall velocity related empirically or theoretically to drop of diameter D [Eq.(3.68)]. It is evident from (4.8)–(4.10) that measurements of the drop size distribution $N(D)$ can be used to determine a set of values $(Z, q_r, \mathfrak{R})$. For rainfall of a given type in a given climatological setting, the curves of log Z vs. log $\mathfrak{R}$ and log Z vs. log ρq_r are usually linear and therefore yield empirical relationships of the form

$$Z = \tilde{a}\mathfrak{R}^{\tilde{b}} \tag{4.11}$$

$$Z = \tilde{a}_1(\rho q_r)^{\tilde{b}_1} \tag{4.12}$$

where $\tilde{a}$, $\tilde{b}$, $\tilde{a}_1$, and $\tilde{b}_1$ are positive constants derived from the slopes and intercepts of the log–log plots. From (4.9) and (4.10), it is evident that the ratio $\mathfrak{R}/(\rho q_r)$ is the mass-weighted particle fall speed $\hat{V}$ defined by (3.69). The relationships (4.11) and (4.12) imply that this ratio is a function of Z of the form

$$\hat{V} = \tilde{a}_2 Z^{\tilde{b}_2} \tag{4.13}$$

where $\tilde{a}_2$ and $\tilde{b}_2$ are constants. Sometimes a factor accounting for the variable density of air through which the particle is falling is included in the fall speed formula.[90] The relationships (4.11)–(4.13) are used for estimating rainfall rate, rain mixing ratio, and rain fall speed from radar reflectivity measurements. Similar relationships can be obtained for snow. The values of the constants should be determined from particle size measurements made within the particular type of precipitation being studied.

To be able to use a Z–$\Re$ relation like (4.11), the equivalent radar reflectivity factor Z_e obtained from the observed radar data must first be converted to an appropriate value of Z. Several types of sampling problems typically make this conversion difficult: (i) The composition of the particles (liquid or ice) must be assumed. This problem is hard to overcome when the beam is partially filled with liquid and ice particles, contains melting ice, or is partially filled by ground targets. (ii) Because of both the curvature of the earth and the elevation angle of the beam, the center of the radar beam is typically found at higher altitude the greater the range. The radar thus indicates a value of Z for a radar resolution volume $\mathcal{V}_{res}$ located some distance above the earth's surface, whereas the drop size distribution measurements are usually made at a point on the surface. The particle size distribution may undergo significant evolution as a result of diffusional and collectional growth, evaporation, breakup, or sedimentation, which may occur in the population of raindrops between the time the drops are in the radar resolution volume and when they finally reach the earth's surface [recall Eq. (3.52)]. This problem is greatly exacerbated if the radar beam at low elevation angles is blocked by an intervening mountain. In such a case, the precipitation at lower altitudes behind the mountain cannot be observed, and a vertical profile of reflectivity must be assumed for the precipitation below the lowest elevation angle that is not blocked.[91] (iii) According to (4.6), Z_e is computed from the returned power $\overline{P_r}$ under the assumption that the distributed targets completely fill the resolution volume $\mathcal{V}_{res}$ at a given range. The returned power may therefore be underestimated if the resolution volume is not filled completely. This problem becomes more likely the greater the range since the beamwidth angles in (4.5) are fixed. These sampling problems lead to an uncertainty of about a factor of 2 (or more in the case of topographic shielding) in estimating rain from Z–$\Re$ relations based on drop size distribution measurements. Some cancellation of errors can be obtained by integrating radar measurements over long times or large areas.[92] The accuracy of Z–q_r and Z–$\hat{V}$ relations is less known but probably similar.

4.2.2.2 Rain Gauge Method

Although the particle size methodology described above for determining precipitation rates from radar data is elegant in that it is developed around the physical relationship between particle size distribution and radar reflectivity, it is limited by the uncertainties mentioned at the end of the preceding subsection.

[90] See Foote and DuToit (1969) or Beard (1985).

[91] The assumed profile could be based on a climatology of the vertical profile of reflectivity. For a discussion of the problem of shielding by mountains, see Joss and Waldvogel (1990).

[92] For further details see Joss and Waldvogel (1990) or Austin (1987).

To avoid these problems, one can directly relate surface rain gauge and radar-measured values of Z_e.

Rain gauge data can be used to obtain the probability density function $P(\Re)$, which is defined such that $P(\Re)\,d\Re$ is the probability that if rain is falling, its rate (in mm h^{-1}) is between $\Re$ and $\Re + d\Re$. For the same population of rain-producing clouds, radar data can be used to determine another probability density function $P(Z_e)$, which is defined such that $P(Z_e)\,dZ_e$ is the probability that if rain (and hence radar echo) is present at a given range from the radar, the equivalent radar reflectivity is between Z_e and $Z_e + dZ_e$. The probability density function for the equivalent radar reflectivity can be determined for an interval of range centered at r. Let the sum of the area covered by echo within this annular region be $A_E(r)$. Let the area covered by those echoes with reflectivity between Z_e and $Z_e + dZ_e$ be $A(Z_e, r)$. Then the probability density function $P(Z_e)$ for the region centered at r is given by

$$P(Z_e)\big|_r dZ_e = \frac{A(Z_e, r)}{A_E(r)} \tag{4.14}$$

It can be shown[93] that since Z_e and $\Re$ are functionally related, the correct transformation of one into the other will produce equal probabilities, such that

$$P(\Re_i)\,d\Re = P(Z_{ei})\,dZ_e \tag{4.15}$$

where the pairs $(Z_{ei}, \Re_i)$ define the Z_{ei}–$\Re_i$ relationship. It follows also that

$$\int_{\Re_i}^{\infty} P(\Re)\,d\Re = \int_{Z_{ei}}^{\infty} P(Z_e)\,dZ_e \tag{4.16}$$

The expression in (4.14) can be substituted in the right-hand integral. If rain gauges located within the annular zone centered at range r from the radar are used to derive $P(\Re)|_r$, which is the probability density function for $\Re$ that applies specifically in this zone, then this empirical probability density can be substituted for the integrand on the left-hand side of (4.16). The Z_e–$\Re$ relationship is then obtained by finding the pairs of lower limits $(Z_{ei}, \Re_i)$ that make the integrals equal. Thus, the empirical probability density functions for Z_e and $\Re$ can be used to obtain a Z_e–$\Re$ relationship that applies at a given range from the radar. By repeating the process for various ranges, Z_e–$\Re$ relationships can be found that apply throughout the field of view of the radar. This technique thus allows one to relate Z_e and $\Re$ without having to account for radar-beam geometry or differences between Z_e and Z.

4.2.3 Estimating Areal Precipitation from Radar Data

One important use of radar data is to estimate rainfall over an area covered by radar observations. A straightforward way to make this estimate is to apply a

[93] See Calheiros and Zawadzki (1987).