constants at radar wavelengths for different targets

	Dielectric constant $\|K\|^2$
)	0.93
	0.176
y ρ_s,	$\approx 0.205\ (\rho_s/\rho_i)^2$

nowflakes contain a lot of air and have smaller dielectric
ing a lot larger, they also have much larger reflectivity than
e mass (recall that the radar reflectivity factor defined in
When these two effects are combined, we find that snow-
dar energy than solid ice particles of the same mass.
bout the nature of the target, or whether it is a Rayleigh
ew quantity called the equivalent reflectivity factor Z_e such

$$\underbrace{\frac{0.55^2 10^{-18}\pi^7 c}{24\log_e(2)}}_{\text{Constants}} \underbrace{\frac{P_t \tau D_a^2}{\lambda^4}}_{\substack{\text{Radar}\\ \text{parameters}}} \underbrace{\frac{T(0,r)^2}{r^2}}_{\text{Path}} \underbrace{\|K_w\|^2 Z_e}_{\substack{\text{Target}\\ \text{properties}}}, \tag{3.4}$$

constant of liquid water (0.93). If attenuation is negligible or
lent reflectivity factor Z_e that is measured by the radar. Note
made of liquid water and are small compared with the radar

ty factor of targets can span many orders of magnitude. For
typical reflectivity factor of 0.01 mm^6/m^3, while the hail core
flectivity factors exceeding 10^6 mm^6/m^3. For convenience, we
y factors in units of decibels (dB, or tenths of orders of
that

$$\text{dBZ} = 10\log_{10}(Z). \tag{3.5}$$

nderstorm would have a reflectivity factor of $10\log_{10}(10^6) =$
factor of the liquid cloud would be $10\log_{10}(0.01) = -20$ dBZ.
lue does not mean a negative reflectivity factor, but simply a
1 mm^6/m^3 (see Box 3.1). On most radar displays, the quantity
ivity" is the equivalent reflectivity factor deduced by the radar
he intensity scale having dBZ units (Fig. 3.1). A good way to
with the meaning of the dBZ values shown on radar displays is
rovided in Table 3.2.

Units and symbols used in the radar meteorology literature can be confusing. First the radar reflectivity factor Z and similar quantities have unusual units of mm^6/mm^3. Then, we do not generally express the reflectivity in linear Z (or mm^6/mm^3) units, but instead in "dBZ," or $10\log_{10}(Z/(1\ mm^6/m^3))$. Complications will grow when we later introduce dual-polarization variables. One of these new variables is "differential reflectivity" Z_{dr}; it is only a difference of two reflectivity measurements when one uses logarithmic units, but is strictly speaking a ratio of reflectivity in linear units. But then we nevertheless display and express it in dB. The possibility for confusion becomes very real when one starts using these symbols in equations: should we use the linear or the dB value? Some authors are trying to resolve the ambiguity by giving lowercase symbols (e.g., z) to linear values, and uppercase symbols (e.g., Z) to dB values. It is being resisted by meteorologists and traditionalists (both of which I am) because we are familiar with Z as a linear quantity in equations, and because z is already commonly used for height in meteorology, introducing additional confusion. Hence, from here onward, I will use Z and other reflectivity-based variables in equations in linear units. Be aware that others may not use a similar convention.

In parallel, one should always be careful whether reflectivity values should be used and manipulated in their linear or logarithmic forms. See Appendix A.2 for a more detailed discussion on this issue.

3.3 Reflectivity factor and rain rate

Consider the following two equations for the reflectivity factor Z and the rainfall rate R:

$$Z = \int_0^\infty N(D) D^6 dD \tag{3.6}$$

and

$$R = \frac{\pi}{6}\int_0^\infty N(D) D^3 w_r(D) dD. \tag{3.7}$$

In these two equations, $N(D)$ is the number of raindrops per unit volume, also referred to as the raindrop size distribution, and $w_r(D)$ is the fall speed of a raindrop of diameter D. The drop size distribution varies with rainfall rate and depends on the processes influencing precipitation growth. Hence, there exists no mathematical function linking Z and R. For example, it can be easily shown that one 2-mm diameter drop falling at 7 m/s has a reflectivity factor similar to 64 drops of 1-mm diameter drops falling at 4 m/s, and yet the

Table 3.2. Typical values of equivalent radar reflectivity factors from different targets

Target type	Z_e (dBZ)
Light drizzle; insects	0
Moderate drizzle; a few raindrops; light snow; migrating birds	10
Light rain or moderate snow, typical of widespread precipitation (1 mm/h)	25
Moderate rain, strong for widespread precipitation (5 mm/h)	35
Heavy rain from a convective shower (20 mm/h)	45
Hail or very heavy rain, peak of thunderstorms (100+ mm/h)	55
Moderate to severe hail	>60

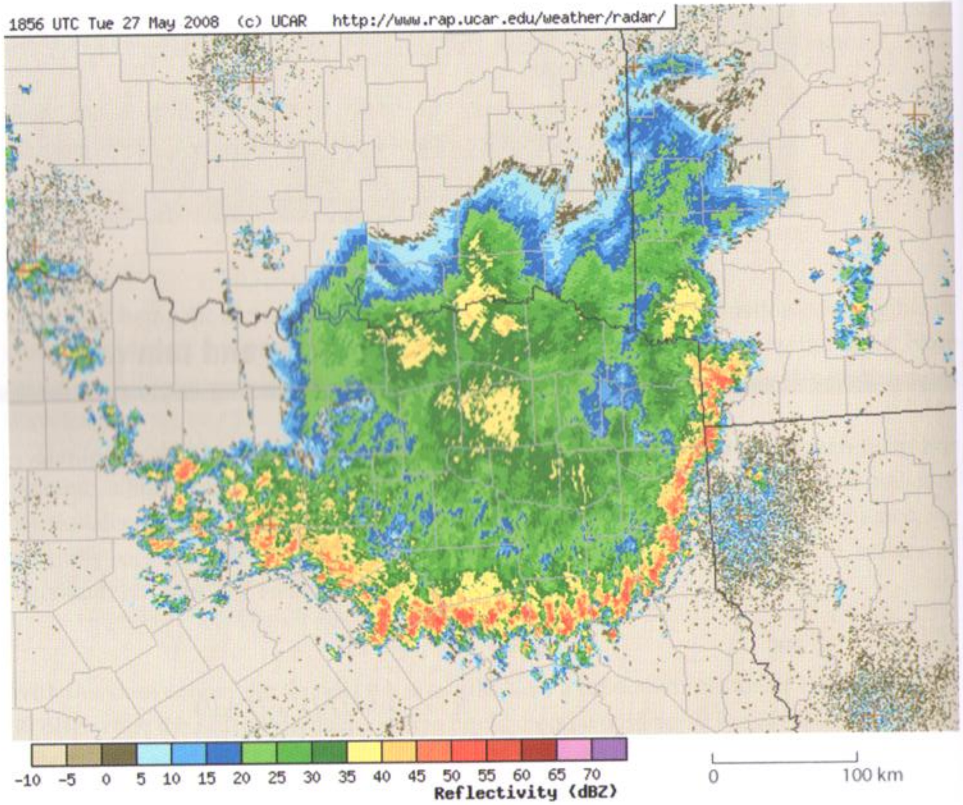
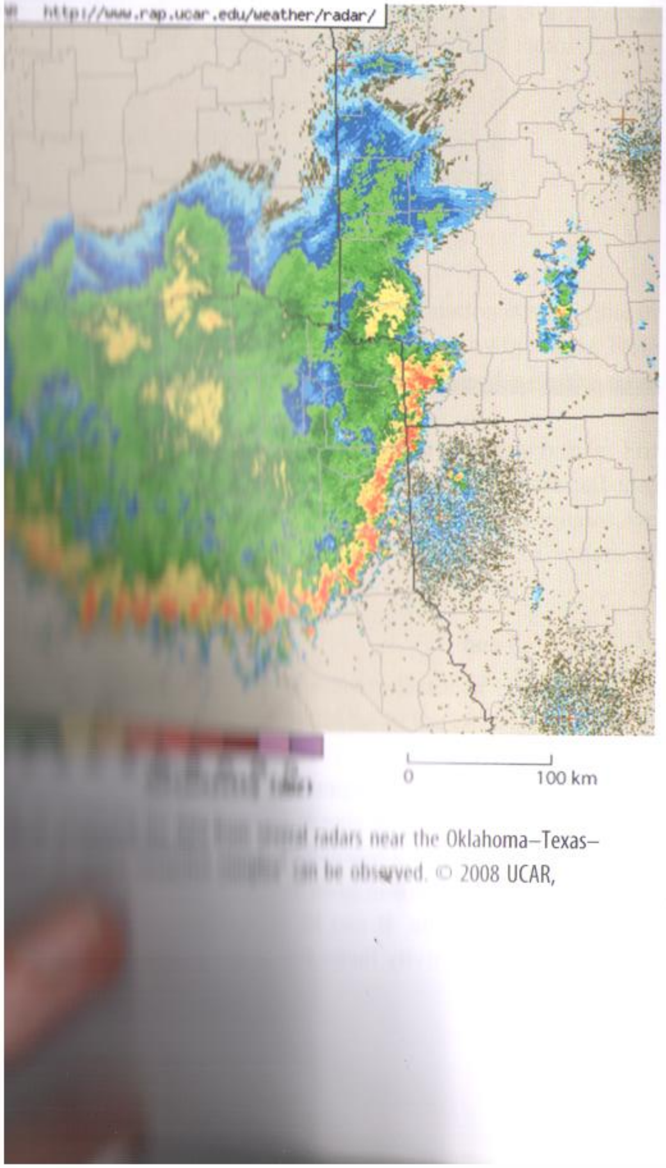


Figure 3.1. Reflectivity composite image made by combining the data from several radars near the Oklahoma–Texas–Arkansas border in the United States. A "smiley convective complex" can be observed. © 2008 UCAR, used with permission.

	Z_e (dBZ)
	0
few raindrops; light snow;	10
te snow, typical of widespread m/h)	25
g for widespread precipitation (5 mm/h)	35
nvective shower (20 mm/h)	45
in, peak of thunderstorms (100+ mm/h)	55
ail	>60



[illegible] radars near the Oklahoma–Texas–
[illegible] can be observed. © 2008 UCAR,

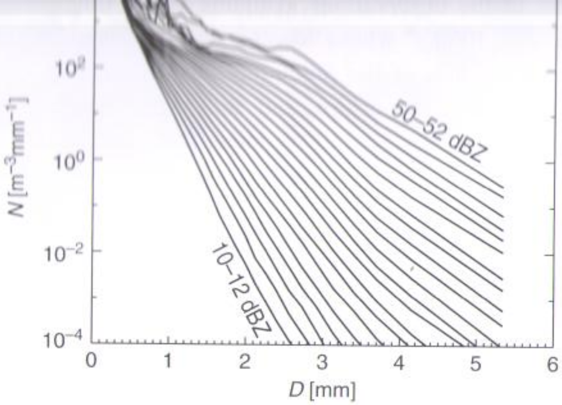


Figure 3.2. Average drop size distribution for all 2-dB reflectivity factor intervals between 10–12 and 50–52 dBZ computed from 5 years of observations in Montreal, Canada. Republished with permission of the AMS from Lee and Zawadzki (2005b); permission conveyed through Copyright Clearance Center, Inc.

two rainfall rates are very different. Because of the D^6 weighting on Z, reflectivity factor is dictated by the larger targets and is relatively unaffected by the smaller drops, even though those contribute a large fraction of the rainfall.

However, on average, drop size distributions vary systematically with reflectivity (Fig. 3.2) and precipitation intensity. This fact has allowed us to derive Z–R relationships that can be used to convert reflectivity factors into rainfall rates. For example, Marshall and Palmer (1948) fitted exponential functions through their drop size distributions and obtained

$$N(D) = 8000\,\mathrm{e}^{(-4.1R^{-0.21}D)}, \tag{3.8}$$

where $N(D)$ is in m^{-3}/mm, R is in mm/h, and D is in mm. Using data on the fall speed of drops as a function of diameter (e.g., Gunn and Kinzer 1949), one gets

$$Z = 300R^{1.5}, \tag{3.9}$$

a relationship Joss and Waldvogel (1970) also found from their disdrometer data. There exist many Z–R relationships, most of them power laws of the form $Z = aR^b$ as in (3.9) but with different a and b coefficients. These change depending on the dynamical and microphysical processes controlling precipitation formation, and hence vary a little from one region to another. For example, precipitation generated by the warm rain process such as drizzle or shallow showers results in smaller drops; hence, areas that receive more warm rain, such as coastal or tropical regions, will have climatological Z–R relationships with smaller a coefficients (a weaker reflectivity will be observed for the same rain rate because of the smaller drops). Examples of other Z–R relationships include the influential Marshall–Palmer Z–R relationship

$Z = 200R^{1.6}$ originally derived in the early 1950s from (3.8) but adjusted to better fit the observations available at the time, and the WSR-88D default relationship $Z = 300R^{1.4}$ whose lower exponent makes it more suitable in regions where a greater fraction of the precipitation is from deep convection. In Chapter 9, we will revisit Z–R relationships and drop size distributions in the context of trying to properly estimate rainfall by radar.

3.4 Radar products

Data collected by radar must be displayed if people are to interpret them. Weather surveillance radars collect data in three spatial dimensions and in time, and those must be rendered on 2-D displays. Some data manipulations or transformations are therefore required. In addition, data such as reflectivity may be further processed in order to extract more information. The results of these manipulations are known as radar products.

As seen in Chapter 2, the most common scanning strategy of weather surveillance radars is to perform several scans in azimuth or PPIs, each at a different elevation angle. The simplest product is hence a PPI display of reflectivity or Doppler velocity at a specific elevation angle. One of the disadvantages of a PPI display is that there is a systematic change in the observation height with range: at close range, one observes echoes near the surface, while at far range echoes originate from much higher in the atmosphere. A way to compensate for this is to build a constant altitude PPI, or CAPPI (Fig. 3.3). A CAPPI is

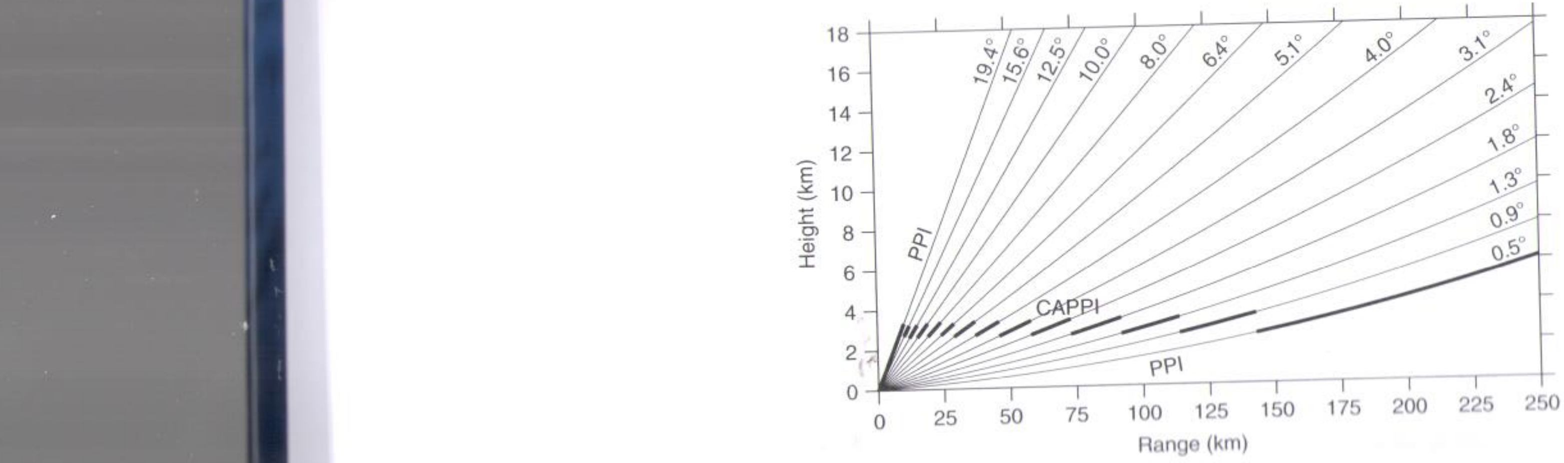


Figure 3.3. Illustration of the generation of a simple 3-km CAPPI. Thin lines represent the PPIs available while the thick lines highlight the elevations from which data are used to make the CAPPI. More sophisticated CAPPIs can also be made by interpolating data from two elevation angles for each pixel.